Transversely Anisotropic Curved Optical Fibers: Variational Analysis of a Nonstandard Eigenproblem

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Abstract —A new variational functional is introduced for the analysis of curved open and closed waveguides. The theory is based on the variational principle for nonstandard eigenvalue problems, recently applied for straight anisotropic fibers. The present method is valid for arbitrary waveguide cross section and arbitrary radius of curvature for closed waveguides, but for open guides, the radius should be large enough because the method predicts the real part of the propagation constant, not the imaginary part, which gives the attenuation in curved open structures. The dielectric medium can be homogeneous or nonhomogeneous with transverse and/or longitudinal anisotropy. As an example of the method, curved isotropic and anisotropic single-mode fibers with two different kinds of anisotropy models are studied. The analysis includes field distributions, changes in the dispersion curves due to reformed geometry, and birefringence characteristics in curved anisotropic fibers.

I. Introduction

THE BENDING of an optical fiber or an open dielectric waveguide has been proved to cause radiation loss, change of the real part of the propagation constant [1], [2], and birefringence, studied particularly in the single-mode optical fibers [3]. The state of the polarization and the field distribution are also modified by the curvature [4], [5]. The loss can be divided into two parts: the pure bending loss due to the uniform curvature and the transition loss related to the mode conversion at the beginning of a bend [6]. The attenuation is proportional to $\sqrt{a/R_0}\exp(-CR_0)$, with C depending on the propagation parameters [1], [7], whereas the phase correction has a $(a/R_0)^2$ dependence for waveguides of symmetrical cross section [1]. R_0 refers to the radius of the bend and a to the characteristic dimension of the waveguide, e.g. the radius of the core in the optical fiber. The bending-induced birefringence is a stress effect [3] which depends on the outer radius d of the waveguide according to $(d/R_0)^2$. Under the bending the outer portion of the fiber cross section is in tension, which, then, presses laterally on the inner portion, which is in compression [8]. This stress modifies the refractive index of the fiber material in a very complicated way [9] and thus generates birefringence. However, there has also been an attempt to analyze the birefringence purely from the geometrical origin [10]. Field deformation and the polarization state in a curved slab waveguide [1], [11], in a rectangular waveguide [11], [12], and in a curved round fiber [4], [5], [13] have been considered. One important result is that in a fiber the polarization states of all modes except the $HE_{1\mu}$ modes changes due to fiber axis curvature [4], [13].

A great deal of effort has been expended on various analytical and numerical methods for the open curved waveguides. The following review introduces some of these studies in the literature. Most of the methods, e.g. [1], stand on the assumption of a large radius of curvature and thus make use of perturbational technique. An approximate eigenvalue equation for a slab waveguide has been derived [11]. Other methods for dielectric planar and rectangular waveguides include perturbational analysis [14], [15] and the straight waveguide approximation [2], [16]. Spectral expansion techniques [14, [17]–[20] as well as beam propagation methods [21], [22] can be applied to general open waveguides. A bent open waveguide can be considered a radiating antenna [23], [24] or a straight waveguide with a modified index of refraction through a conformal transformation [12], [26], [27]. An exact numerical analysis in the toroidal coordinate system has also been introduced [28]. Expansions of the slab waveguide theory to include the optical fiber have been carried out [29], [30]. Geometrical optics [31] and analytical loss formulas of the fiber, such as [32], are also available in the literature. One group of reports applies coupled-mode theory to estimate mode conversion in a bent waveguide [8], [9], [33], [34]. The variational technique has received scant attention among the various methods dealing with curved waveguides. There seems to be only one study based on this technique [35]. The purpose of this paper is to introduce a new variational method applicable to curved open and closed waveguides. The dielectric medium of the waveguide can be inhomogeneous and anisotropic.

Wave propagation in a waveguide can be governed by the following abstract equation [36]:

$$L(\lambda)f=0 \tag{1}$$

where $L(\lambda)$ and λ stand for the operator and the eigenvalue parameter of the problem, respectively. The problem

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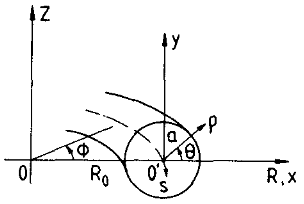

Fig. 1. A uniformly bent anisotropic waveguide and the related coordinate systems.

is called nonstandard if $L(\lambda)$ is a nonlinear function of λ; otherwise it is called standard. Boundary conditions associated with closed waveguides or dielectric boundaries can be hidden in the formula (1) with a suitable choice of the field f or they can be taken into account by an additional equation. Equation (1) can be solved by a variational method, provided that the inner product

$$(f, L(\lambda)f) = 0 \tag{2}$$

exists. If the operator L is self-adjoint with respect to this inner product, (2) possesses stationary roots for λ, as was proved in [37]. For the eigenvalue λ one can take any possible geometrical or physical parameter involved in the problem. Stationary roots can also be obtained for a given new parameter which has been derived from the old ones.

In Section II, the formulation of a curved waveguide in terms of the longitudinal fields is seen to lead to a nonstandard eigenproblem (1). With a definition of a proper inner product (2), a stationary functional is derived. In Section III this functional is shown to be a generalization of the former functional for the straight waveguide [36]. In Section IV, the present theory is applied to a bent, weakly guiding step-index fiber. For the trial fields, asymptotic fields derived under the weakly guiding assumption and for large radius of curvatures are used. In Section V, a curved anisotropic fiber with two different kinds of anisotropy models is analyzed. Section VI contains the conclusions of this paper.

II. Theory

We consider a bent anisotropic waveguide with a radius of curvature R_0 in the global cylindrical coordinate system $(0, R, \phi, Z)$, Fig. 1. The guiding direction is along the axis of the waveguide, which is the s axis of the local toroidal coordinate system $(0', \rho, \theta, s)$ or $(0', x, y, s)$. The two coordinate systems are related by the following equations:

$$R = R_0 + \rho\cos\theta \qquad x = \rho\cos\theta \tag{3}$$

$$Z = \rho\sin\theta = y \tag{4}$$

$$\phi = -s/R_0. \tag{5}$$

In each coordinate system, the unit vector in the i direction is denoted by $\boldsymbol{u}_i$. The field components in the two coordinate systems are related by

$$E_R = E_x \qquad E_Z = E_y \qquad E_\phi = -E_s \tag{6}$$

$$E_x = E_\rho\cos\theta - E_\theta\sin\theta \qquad E_y = E_\rho\sin\theta + E_\theta\cos\theta. \tag{7}$$

We assume a field formulation $\boldsymbol{E}(R,Z)\exp(-j\nu\phi)$ or $\boldsymbol{E}(\rho,\theta)\exp(+j\beta s)$ with time dependence $\exp(j\omega t)$ omitted. Curl operators in the local (l) and global (g) coordinate systems are related to each other by

$$(\nabla\times\boldsymbol{A})_g = (\nabla\times\boldsymbol{A})_l + A_\phi\boldsymbol{u}_\phi/R \tag{8}$$

where subscripts g and l stand for the transverse differential operator obtained by setting $\partial/\partial\phi$ (g) or $\partial/\partial s$ (l) equal to zero in the two coordinate systems, respectively. The gradient operator is invariant with respect to the coordinate system: $(\nabla f)_g = (\nabla f)_l$. For the partial derivatives we have $\partial/\partial R = \cos\theta\partial/\partial\rho - \sin\theta\partial/\rho\partial\theta$ and $\partial/\partial Z = \cos\theta\partial/\rho\partial\theta + \sin\theta\partial/\partial\rho$.

The waveguide is modeled by a symmetric dielectric dyadic

$$\boldsymbol{\epsilon}_k(\boldsymbol{\rho}) = \boldsymbol{\epsilon}(\boldsymbol{\rho}) + \epsilon_\phi(\boldsymbol{\rho})\boldsymbol{u}_\phi\boldsymbol{u}_\phi \tag{9a}$$

where $\boldsymbol{\epsilon}(\boldsymbol{\rho}) = \epsilon_v(\boldsymbol{\rho})\boldsymbol{u}_v\boldsymbol{u}_v + \epsilon_w(\boldsymbol{\rho})\boldsymbol{u}_w\boldsymbol{u}_w$ is a two-dimensional dyadic in the transverse plane xy with a position vector $\boldsymbol{\rho}$ and orthogonal vectors $\boldsymbol{u}_v$, $\boldsymbol{u}_w$ in that plane. The dielectric medium will be assumed lossless. In open guides the bend generates compressional strain, which changes permittivity. For an optical fiber these fractional increases in those directions, x and y, which contribute to the propagation characteristics of the waveguide have been estimated to be of the order of $0.0015(d/R_0)^4\epsilon$ and $0.018(d/R_0)^4\epsilon$ [34], where d is the outer diameter of the fiber. To add these additional terms $\delta\epsilon x, \delta\epsilon y$ to the formula (9a) results in the following permittivity matrix:

$$[\epsilon_k(\boldsymbol{\rho})] = \begin{bmatrix} \epsilon_v + \delta\epsilon_x\cos^2\alpha + \delta\epsilon_y\sin^2\alpha & (\delta\epsilon_y - \delta\epsilon_x)\sin\alpha\cos\alpha & 0 \\ (\delta\epsilon_y - \delta\epsilon_x)\sin\alpha\cos\alpha & \epsilon_w + \delta\epsilon_x\cos^2\alpha + \delta\epsilon_y\sin^2\alpha & 0 \\ 0 & 0 & \epsilon_\phi \end{bmatrix} \tag{9b}$$

which is still symmetric. α is the angle between $\boldsymbol{u}_x$ and $\boldsymbol{u}_v$ vectors.

As in [36], we derive equations for the longitudinal fields. We start by writing the guided fields in the global coordinate system as $\boldsymbol{E}(R,Z)\exp(-j\nu\phi) = (\boldsymbol{e}(R,Z) + e(R,Z)\boldsymbol{u}_\phi)\exp(-j\nu\phi)$ and $\boldsymbol{H}(R,Z)\exp(-j\nu\phi) = (\boldsymbol{h}(R,Z) + h(R,Z)\boldsymbol{u}_\phi)\exp(-j\nu\phi)$ and then insert them in Maxwell's equations. After some algebra we are left with the following equations

$$\nabla\times\boldsymbol{e} + j\omega\mu h\boldsymbol{u}_\phi = 0 \tag{10}$$

$$\nabla\times\boldsymbol{h} - j\omega\epsilon_\phi e\boldsymbol{u}_\phi = 0 \tag{11}$$

$$\nabla(eR)\times\boldsymbol{u}_\phi + j\nu\boldsymbol{e}\times\boldsymbol{u}_\phi + j\omega\mu R\boldsymbol{h} = 0 \tag{12}$$

$$\nabla(hR)\times\boldsymbol{u}_\phi + j\nu\boldsymbol{h}\times\boldsymbol{u}_\phi - j\omega R\boldsymbol{\epsilon}\cdot\boldsymbol{e} = 0. \tag{13}$$

Here $\boldsymbol{u}_\phi$ is the axial unit vector, whence for the transverse fields we have $\boldsymbol{u}_\phi \cdot \boldsymbol{e} = 0$ and $\boldsymbol{u}_\phi \cdot \boldsymbol{h} = 0$. The curl operator denotes a transverse differential operator. From (12) and (13) we eliminate the transversal field vectors $\boldsymbol{e}$ and $\boldsymbol{h}$:

$$\boldsymbol{e} = -jk_c^{-2} \cdot \left[\nu \nabla (Re)/R^2 + \omega\mu \nabla (Rh) \times \boldsymbol{u}_\phi / R\right] \quad (14)$$

$$\boldsymbol{h} = j\boldsymbol{u}_\phi \times \boldsymbol{k}_c^{-2} \cdot \left[-\omega \boldsymbol{\epsilon} \cdot \nabla (Re)/R + \nu \boldsymbol{u}_\phi \times \nabla (Rh)/R^2\right]. \quad (15)$$

$$Lf = \begin{bmatrix} \omega \boldsymbol{u}_\phi \cdot \nabla \times \left[\boldsymbol{k}_c^{-2} \cdot \boldsymbol{\epsilon} \cdot \nabla (eR) \times \boldsymbol{u}_\phi / R\right] - \omega \epsilon_\phi e - \nu \boldsymbol{u}_\phi \cdot \nabla \times \left[\left(\boldsymbol{k}_c^{-2} \overset{\times}{\times} \boldsymbol{u}_\phi \boldsymbol{u}_\phi\right) \cdot \nabla (Rh)/R^2\right] \\ \nu \boldsymbol{u}_\phi \cdot \nabla \times \left[\boldsymbol{k}_c^{-2} \cdot \nabla (Re)/R^2\right] - \omega\mu h + \omega\mu \boldsymbol{u}_\phi \cdot \nabla \times \left[\boldsymbol{k}_c^{-2} \cdot \nabla (hR) \times \boldsymbol{u}_\phi / R\right] \end{bmatrix}. \quad (23)$$

Here $\boldsymbol{k}_c^{-2}$ is the inverse of the two-dimensional dyadic

$$\boldsymbol{k}_c^2 = \omega^2 \mu \boldsymbol{\epsilon} - (\nu/R)^2 \boldsymbol{E} \quad (16)$$

and $\boldsymbol{E}$ is the two-dimensional unit dyadic: $\boldsymbol{E} = \boldsymbol{I} - \boldsymbol{u}_\phi \boldsymbol{u}_\phi$. The inverse dyadic can be defined as [36]

$$\boldsymbol{k}_c^{-2} = \left(\boldsymbol{k}_c^2 \overset{\times}{\times} \boldsymbol{u}_\phi \boldsymbol{u}_\phi\right) / \mathrm{spm}\left(\boldsymbol{k}_c^2\right). \quad (17)$$

The definition of the double cross product here is as follows: $(\boldsymbol{ab}) \overset{\times}{\times} (\boldsymbol{cd}) = (\boldsymbol{a} \times \boldsymbol{c})(\boldsymbol{b} \times \boldsymbol{d})$ and spm( ) is the two-dimensional determinant function (sum of principal minors of the three-dimensional dyadic) defined by $\mathrm{spm}\, \boldsymbol{A} = \boldsymbol{A} \overset{\times}{\times} \boldsymbol{A} : \boldsymbol{I}/2$. Inserting (16) into (17) and using the above definition for spm( ) gives us the inverse dyadic

$$\boldsymbol{k}_c^{-2} = k_{cv}^{-2} \boldsymbol{u}_v \boldsymbol{u}_v + k_{cw}^{-2} \boldsymbol{u}_w \boldsymbol{u}_w \quad (18)$$

where the components are

$$k_{ci}^{-2} = \left(\omega^2 \mu \epsilon_i - (\nu/R)^2\right)^{-1}, \quad i = v, w. \quad (19)$$

The equations for the longitudinal fields can now be written by substituting (14) and (15) into (10) and (11). After rearranging terms and using vector analysis, we obtain

$$\nabla \times \left\{\boldsymbol{u}_\phi \times \boldsymbol{k}_c^{-2} \cdot \left[-\omega \boldsymbol{\epsilon} \cdot \nabla (eR)/R + \nu \boldsymbol{u}_\phi \times \nabla (Rh)/R^2\right]\right\} - \omega \epsilon_\phi e \boldsymbol{u}_\phi = 0 \quad (20)$$

$$\nabla \times \left\{\boldsymbol{k}_c^{-2} \cdot \left[\nu \nabla (eR)/R^2 + \omega\mu \nabla (hR) \times \boldsymbol{u}_\phi / R\right]\right\} - \omega\mu h \boldsymbol{u}_\phi = 0. \quad (21)$$

These equations form the basis for the present theory. In order to apply the variational method, we have to define a proper inner product (2) and the operator $L(\lambda)$ in (1) which is self-adjoint with respect to that inner product. These requirements can be satisfied by the following definitions. The unknown fields are longitudinal fields (e, h), which we denote here by f as in (1) and (2), and define the following inner product $(\cdot, \cdot)$:

$$(f_1, f_2) = \int_0^\infty (e_1, h_1) \cdot \begin{pmatrix} e_2 \\ h_2 \end{pmatrix} dS. \quad (22)$$

Next we assume the boundary of the waveguide to be a perfect conductor, and assume the fields to obey the corresponding boundary conditions. This assumption simplifies the derivation of the functional, but causes a small error for the propagation parameters when the functional is applied to the open waveguides. This will be discussed later.

The operator L operates on the pair of scalar functions (e, h) according to (23). To verify that the above definitions lead to a self-adjoint formulation, we form the inner product according to (22)

$$\begin{aligned}(f_1, Lf_2) = &-\omega \int \{\epsilon_\phi e_1 e_2 + \mu h_1 h_2\}\, dS \\ &+ \omega\mu \int \{(\nabla (h_1 R) \times \boldsymbol{u}_\phi / R) \\ &\cdot \boldsymbol{k}_c^{-2} \cdot (\nabla (h_2 R) \times \boldsymbol{u}_\phi / R)\}\, dS \\ &+ \omega \int \{((e_1 R) \times \boldsymbol{u}_\phi / R) \\ &\cdot \boldsymbol{k}_c^{-2} \cdot \boldsymbol{\epsilon} \cdot (\nabla (e_2 R) \times \boldsymbol{u}_\phi / R)\}\, dS \\ &+ \nu \int \{\nabla (h_1 R)/R \cdot (\boldsymbol{u}_\phi \times \boldsymbol{k}_c^{-2}) \\ &\cdot \nabla (e_2 R)/R^2 + \nabla (h_2 R)/R^2 \\ &\cdot (\boldsymbol{u}_\phi \times \boldsymbol{k}_c^{-2}) \cdot \nabla (e_1 R)/R\}\, dS = 0 \end{aligned} \quad (24)$$

where the divergence terms, reduced to line integrals at the boundary of the waveguide, do not contribute because of the assumed requirement for the fields. To be self-adjoint, (24) should be symmetric in 1 and 2. The first term is clearly symmetric and so are the second and third terms, provided that the dyadic $\boldsymbol{\epsilon}$ is symmetric, as was assumed in (9a) and (9b). The last terms are not symmetric; instead they form a symmetric pair. As a conclusion, L is self-adjoint and we can apply the functional (2) written for this special problem in (24).

The operator L, defined in (23) contains the dyadic $\boldsymbol{k}_c^{-2}$, which is a complicated function of all the physical parameters involved in the problem. Bearing this in mind, there is no hope of solving the functional (24) explicitly for any of the parameters. This is evidently true also for any possible geometrical parameters, which can be included in the permittivity dyadic $\boldsymbol{\epsilon}$. Thus the eigenvalue equation $Lf = 0$ is of a nonstandard type [37]. To overcome difficulties involved in this kind of problem we proceed analogously to [36] and define new parameters from the old ones and try to solve the functional in terms of these. In fact, forming a two-dimensional dyadic as

$$\begin{aligned}\boldsymbol{M} &= \omega^2 \boldsymbol{k}_c^{-2} / R^2 \\ &= \left(R^2 \mu \epsilon_v - \gamma^{-2}\right)^{-1} \boldsymbol{u}_v \boldsymbol{u}_v + \left(R^2 \mu \epsilon_w - \gamma^{-2}\right)^{-1} \boldsymbol{u}_w \boldsymbol{u}_w \end{aligned} \quad (25)$$

where we have defined a new parameter γ as

$$\gamma = \omega / \nu \quad (26)$$

we can express the functional (24) in a compact form for the parameter ω^2:

$$\omega^2 = \frac{\int \left\{ \nabla(Re)\cdot \boldsymbol{M}\cdot \boldsymbol{\epsilon}\cdot \nabla(Re) + \left(\boldsymbol{u}_\phi \times \nabla(Rh)\right)\cdot \boldsymbol{M}\mu\cdot\left(\boldsymbol{u}_\phi \times \nabla(Rh)\right) - 2\boldsymbol{u}_\phi \times \nabla(Rh)\cdot \boldsymbol{M}\cdot \nabla(Re)/(R\gamma)\right\} dS}{\int \left\{ \epsilon_\phi e^2 + \mu h^2 \right\} dS} \tag{27}$$

where the integration extends over the waveguide cross section. Equation (27) is a stationary functional for ω^2. The above derivation could have been carried out in terms of the transversal fields, whence the functional would have been of the standard form in both ω^2 and β^2. However, the operator L would then be non-self-adjoint with respect to the symmetric inner product applied here and the proper solution would also require the adjoint problem to be taken into account, as was noted in [38] for the case of the straight corrugated waveguide.

The functional (27) is exact for the closed curved waveguides, and can handle an arbitrary radius of curvature. The only requirements for the fields are continuity everywhere and ideal conductor conditions for the fields at the waveguide boundary. The dielectric tensor can be arbitrary provided that it is symmetric. With these comments the above equation is the most general functional for closed bent waveguides and, to the knowledge of the authors, has not been introduced before. For the isotropic waveguide, the functional takes on the simplified form

$$\omega^2 = \frac{\int \left(R^2\epsilon\mu - \gamma^{-2}\right)^{-1}\left\{\epsilon\left(\nabla(Re)\right)^2 + \mu\left(\nabla(Rh)\right)^2 - 2\boldsymbol{u}_\phi\cdot\nabla(Rh)\times\nabla(Re)/(R\gamma)\right\} dS}{\int \left(\epsilon e^2 + \mu h^2\right) dS}. \tag{28}$$

When the waveguide is open, such as in the optical fiber, the functional can be used only as an approximation. This comes from the different boundary conditions. If the open guide is bent, it becomes radiative, and the fields at infinity are nonzero. Then the line integrals must be added in (24), and the functional includes additional terms. However, if we assume the radius of curvature to be sufficiently large, so that the attenuation is very small due to the exponential decay [1], [7], the functional (27) can be applied to estimate the change of the real part of the propagation constant due to the bending. This part is essentially larger than the imaginary part because here the dependence is of the order $(a/R_0)^2$ [1].

III. Testing the Functional: The Radius of Curvature → ∞

As the radius of curvature R_0 in (3) goes to infinity, the torodial coordinate system straightens to the cylindrical one with coordinates ρ, θ, and s. Then the following transformations hold:

$$\nu/R \to \nu/R_0 \to \beta_0 \tag{29}$$

$$R\gamma = R\omega/\nu \to R_0\omega/\nu = \omega/\beta_0 = v_p \tag{30}$$

$$\begin{aligned} \boldsymbol{M} \to\ & R_0^2\left(\mu\epsilon_v - (\nu/\omega R_0)^2\right)^{-1}\boldsymbol{u}_v\boldsymbol{u}_v \\ & + R_0^2\left(\mu\epsilon_w - (\nu/\omega R_0)^2\right)^{-1}\boldsymbol{u}_w\boldsymbol{u}_w \\ & = R_0^2\left\{\left(\mu\epsilon_v - v_p^{-2}\right)^{-1}\boldsymbol{u}_v\boldsymbol{u}_v + \left(\mu\epsilon_w - v_p^{-2}\right)^{-1}\boldsymbol{u}_w\boldsymbol{u}_w\right\} \\ & = R_0^2 \boldsymbol{N} \end{aligned} \tag{31}$$

where v_p refers to the phase velocity of the field and $\boldsymbol{N}$ is a two-dimensional dyadic.

Inserting (29)–(31) into (27) and taking the limit as $R \to \infty$ gives us

$$\omega^2 = \frac{\int_0^\infty \left\{\nabla e\cdot\boldsymbol{N}\cdot\boldsymbol{\epsilon}\cdot\nabla e + (\boldsymbol{u}_s\times\nabla h)\cdot\boldsymbol{N}\cdot\mu(\boldsymbol{u}_s\times\nabla h) - 2(\boldsymbol{u}_s\times\nabla h)\cdot\boldsymbol{N}\cdot\nabla e/v_p\right\} dS}{\int_0^\infty \left\{\epsilon_\phi e^2 + \mu h^2\right\} dS}. \tag{32}$$

This is identical with the functional for the straight open waveguide [35]. The trial fields are now exponentially decaying as $\rho \to \infty$. Unfortunately, the functional in [36] is not written correctly. The coefficient in the last term in the functional should read $-2/v_p$ and not -2.

The same limiting process can be performed for the isotropic guide, whence we obtain

$$\omega^2 = \frac{\int_0^\infty \left(\mu\epsilon - v_p^{-2}\right)^{-1}\left(\epsilon(\nabla e)^2 + \mu(\nabla h)^2 - 2\boldsymbol{u}_s\cdot\nabla h\times\nabla e/v_p\right) dS}{\int_0^\infty \left(\epsilon e^2 + \mu h^2\right) dS} \tag{33}$$

which again is identical with that derived in [36] for the straight isotropic waveguide.

IV. The Bent Isotropic Step-Index Fiber

As a first example we consider a weakly guiding isotropic step-index optical fiber bent by a radius of R_0 (Fig. 1). Here we treat the fiber purely from the geometrical point of view. This means that small changes in dielectric properties of the fiber due to the bending are ignored. Also, the linear birefringence resulting from this modified permittivity is not taken into account. From the optical waveguide theory we adopt the usual notations for the normalized frequency V, normalized propagation constant b, and parameter Δ, the normalized dielectric constant difference between the core and the cladding of the fiber:

$$V = a\sqrt{k_1^2 - k_2^2} = k_2 a\sqrt{\Delta} \tag{34}$$

$$b = \frac{\beta^2 - k_2^2}{k_1^2 - k_2^2} = \frac{\beta^2 - k_2^2}{V^2} a^2 \tag{35}$$

$$\Delta = \frac{k_1^2 - k_2^2}{k_2^2} = \frac{\epsilon_1 - \epsilon_2}{\epsilon_2}. \tag{36}$$

Here, 1 stands for the medium in the core and 2 for that in the cladding. The radius of the core is denoted by a. The propagation constant β is related to the azimuthal index ν and the radius R_0 according to $\beta = \nu/R_0$, which we expand as a power series:

$$\beta^2 = \beta_0^2 + \beta'^2(\alpha/R_0)^2 + \cdots \tag{37}$$

where β_0 denotes the propagation constant of the straight fiber and $\alpha = \beta_0^2 a^3/4$. In the weakly guiding fiber we have $\alpha \approx V^2 a/4\Delta$. The term proportional to $(1/R_0)$ is missing in the expansion. This has been noted in many asymptotic studies of the curved waveguide when the waveguide possesses a symmetric structure with certain symmetry properties of the mode fields, e.g. [1] and [39]. This holds as well for closed waveguides [40] and for symmetric dielectric slab waveguides [1]. The expansion is valid only for large radius of curvature. If the radius becomes small, the convergence is very poor, and the propagation constant has a $R_0^{-2/3}$ dependence [41].

The dielectric function can be written from (36) as

$$\epsilon(\rho) = \epsilon_2(1 + \Delta P(\rho)) \tag{38}$$

where $P(\rho)$ is the pulse function for the step-index fiber: $P(\rho) = 1$ for $\rho \leqslant a$, and $= 0$ for $\rho > a$.

Field deformation in a curved step-index fiber has been solved by a numerical method using Fourier–Bessel series expansions [5]. Analytical equations for the transversal fields have been derived by applying a first-order perturbation theory [4]. In [13] the theory was extended to include radially inhomogeneous fibers and longitudinal fields. Studies based on a Gaussian function approximation for the fields have been published [42], [43]. Field deformation can cause a significant contribution to the radiation loss of the fiber, as was noted in [32]. Here we derive longitudinal fields in the curved fiber starting from (20) and (21), which we take for the isotropic waveguide. The resulting field equations are similar to those in [13].

After rearranging terms in (20) we are left with an equation for the longitudinal field in the curved waveguide:

$$\nabla^2 e - \frac{e}{R^2} + k_c^2 e + \frac{2\nu k^2}{\omega\epsilon R^2 k_c^2}\partial h/\partial Z - \frac{2\nu^2}{R^4 k_c^2}\partial(Re)/\partial R = 0 \tag{39}$$

where $k_c^2 = k^2 - (\beta R_0/R)^2$ and $\nu = \beta R_0$. The corresponding equation for the magnetic field can be obtained by changing e to h and ϵ to $-\mu$ and vice versa. Equation (39) is exact and contains no approximations. To solve it we apply a perturbational method where the solution can be sought as a power series in R_0, assuming

$$e = e_0 + e_1\alpha/R_0 + e_2(\alpha/R_0)^2 + \cdots. \tag{40}$$

By inserting (37) and (40) into (39) and equating powers of R_0 we obtain a set of differential equations for the field e.

The zero-order equation reads

$$\nabla_t^2 e_0 + (k^2 - \beta_0^2)e_0 = 0 \tag{41}$$

where ∇_t^2 denotes the transverse Laplace operator and $k^2 = k_2^2(1 + \Delta P)$. The first-order equation is

$$\nabla_t^2 e_1 + (k^2 - \beta_0^2)e_1 = \frac{8}{a^3(k^2 - \beta_0^2)}\partial e_0/\partial R + \frac{8k^2}{\beta_0\omega\epsilon a^3(k^2 - \beta_0^2)}\partial h_0/\partial Z - \frac{8\rho\cos\theta}{a^3}e_0 \tag{42}$$

and $\epsilon = \epsilon_2\sqrt{1 + P\Delta}$. In the weakly guiding waveguide we have approximately $\beta_0 \approx k$.

By solving (41) we have, for the lowest order HE_{11} mode,

$$e_0 = \begin{cases} J_1(u\rho/a)/J_1(u)\cos(\theta - \theta_0) & \text{for } \rho \leqslant a \\ K_1(w\rho/a)/K_1(w)\cos(\theta - \theta_0) & \text{for } \rho \geqslant a. \end{cases} \tag{43}$$

J and K denote the Bessel and modified Hankel functions, respectively. θ_0 is the polarization angle: $\theta_0 = 0°$ and $\theta_0 = 90°$ correspond to the mode polarized in the x and the y direction, respectively. u and w are the normalized parameters, defined as $u^2 = (k_1^2 - \beta_0^2)a^2$; $w^2 = (\beta_0^2 - k_2^2)a^2$. In terms of the normalized frequency and propagation constants we have $u = V\sqrt{1 - b_0}$ and $w = V\sqrt{b_0}$. The normalized parameter b_0 is the solution of the eigenvalue equation at the limit $\Delta \to 0$: $uJ_1(u)/J_0(u) = wK_1(w)/K_0(w)$ [44].

If we denote the radial dependence of the zero-order solution e_0 by $t(\rho)$, the corresponding magnetic field is

$$h_0 = Ht(\rho)\sin(\theta - \theta_0) \tag{44}$$

where H is the magnetic field coefficient, which can be approximated by $H = \sqrt{\epsilon_2/\mu}$ for the HE_{11} mode under the weakly guiding assumption [44]. The same value for H can also be obtained by optimizing the functional (33) with respect to this parameter, as was demonstrated in [36].

The first-order fields are solutions of (42). They are, with the assumption $\beta_0 \approx k$,

$$e_1 = \begin{cases} \left\{\left(\delta + (\rho/a)^2 u^{-1}\right) J_0 + 2(\rho/a) J_1 u^{-2}\right\}/J_1(u)\cos\theta_0 \\ \quad + \left\{\left(\delta + (\rho/a)^2 u^{-1}\right) J_0 - 2\delta(a/\rho) J_1 u^{-1}\right\}/J_1(u)\cos(2\theta - \theta_0) & \text{for } \rho \leqslant a \\ \left\{\left(\sigma + (\rho/a)^2 w^{-1}\right) K_0 - 2(\rho/a) K_1 w^{-2}\right\}/K_1(w)\cos\theta_0 \\ \quad + \left\{\left(\sigma + (\rho/a)^2 w^{-1}\right) K_0 + 2\sigma(a/\rho) K_1 w^{-1}\right\}/K_1(w)\cos(2\theta - \theta_0) & \text{for } \rho \geqslant a. \end{cases} \tag{45}$$

The error in the field equations due to the above assumption is at most of the order Δ. The arguments $u\rho/a$ and $w\rho/a$ of the special functions in the denominators are omitted. δ and σ are constants derived from the boundary conditions: $\delta = -K_2(w)/uK_0(w)$; $\sigma = J_2(u)/wJ_0(u)$.

Analogously to (44), we write $e_1 = f(\rho)\cos\theta_0 + g(\rho)\cos(2\theta - \theta_0)$. This gives us the magnetic field h_1:

$$h_1 = H(-f(\rho)\sin\theta_0 + g(\rho)\sin(2\theta - \theta_0)). \tag{46}$$

To combine the field classification we write, for the HE_{11x} mode,

$$e = t(\rho)\cos\theta + (\alpha/R_0)(f(\rho) + g(\rho)\cos 2\theta)$$
$$h = H(t(\rho)\sin\theta + (\alpha/R_0) g(\rho)\sin 2\theta) \tag{47}$$

and for the HE_{11y} mode,

$$e = t(\rho)\sin\theta + (\alpha/R_0) g(\rho)\sin 2\theta$$
$$h = -H(t(\rho)\cos\theta + (\alpha/R_0)(f(\rho) + g(\rho)\cos 2\theta)).$$

In the weakly guiding limit we have $\alpha = \beta_0^2 a^3/4 \approx V^2 a/4\Delta$.

The following relations hold between the mode fields:

$$e^y = h^x/H \qquad h^y/H = -e^x \tag{48}$$

where the subscripts x and y denote the HE_{11x} and HE_{11y} modes, respectively.

The dispersion relation in the curved fiber can now be calculated from the stationary functional (28), which we express in terms of the normalized parameters V, b, and Δ. The parameter b is a function of the propagation constant β, so we write

$$b = b_0 + b'(\alpha/R_0 V)^2 + \cdots \approx b_0 + \delta b. \tag{49}$$

Here δb stands for the change of the normalized parameter and is approximated by the second term $b'(Va/4\Delta R_0)^2$ of the asymptotic series. The first-order correction is missing due to the expansion (37). In the weakly guiding limit $\Delta \to 0$, the functional reads, in the local coordinate system,

$$V^2 = \frac{\displaystyle\int_0^\infty \frac{a^2\left(\sqrt{\epsilon_2}\nabla e - \sqrt{\mu}\, \boldsymbol{u}_s \times \nabla h\right)^2}{(P - b_0)F(P, b_0)} \rho\, d\rho\, d\theta}{\displaystyle\int_0^\infty \left(\epsilon_2 e^2 + \mu h^2\right)\rho\, d\rho\, d\theta}$$

$$F(P, b_0) = 1 + (R_0\Delta)^{-1} 2\rho\cos\theta/(P - b_0) - \delta b V^2/(P - b_0). \tag{50}$$

The functional is independent of the wave polarization in the weakly guiding limit for the HE_{11} mode. This is seen by inserting field relations (47) into (50), in which case the integrands remain unchanged. This means that the bend induces the same change in the propagation constants of the orthogonal polarized waves of the HE_{11} mode. This has been observed previously by many authors applying a variety of methods, e.g. [18].

The unknown parameter δb is hidden in the functional in a complicated way. To solve this, we can either apply an asymptotic method or make use of some numerical algorithm. In the latter, we start from a definite (V, b_0) point in the dispersion curve, insert the field expressions, and seek a proper value for δb so that (50) is satisfied. In the former method we equate coefficients of equal powers of (α/R_0) in (50). The lowest order equation is

$$V^2 = \frac{\displaystyle\int_0^\infty \frac{a^2}{(P - b_0)}\left(t' + \frac{t}{\rho}\right)^2 \rho\, d\rho}{\displaystyle\int_0^\infty t^2 \rho\, d\rho}. \tag{51}$$

Here, the prime denotes the first derivative of the function with respect to ρ. This is the functional equation for the straight fiber derived in [36] and gives the corresponding eigenvalue equation by using the longitudinal fields (43). The first-order equation is lacking, which is evident from the asymptotic nature of the propagation constant. The second-order equation gives an expression for the change of the propagation constant δb:

$$\delta b = \frac{\displaystyle V^6 \int_0^\infty M_1(\rho)\rho\, d\rho - V^4 \int_0^\infty \frac{a^2 M_2(\rho)}{(P - b_0)}\rho\, d\rho + V^2 \int_0^\infty \frac{a M_3(\rho)}{(P - b_0)^2}\rho\, d\rho - \int_0^\infty \frac{2M_4(\rho)}{(P - b_0)^3}\rho^3\, d\rho}{\displaystyle a^2 \int_0^\infty \frac{M_4(\rho)}{(P - b_0)^2}\rho\, d\rho} \left(\frac{a}{4\Delta R_0}\right)^2 \tag{52}$$

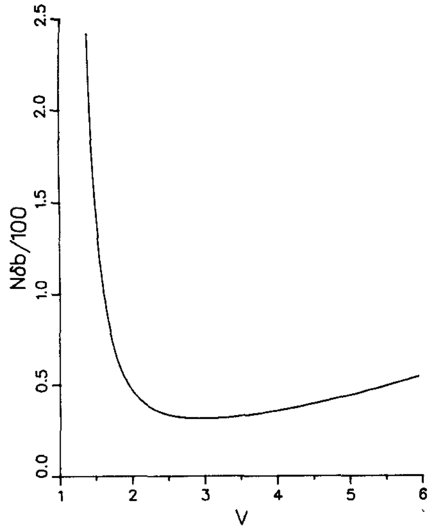

Fig. 2. Change of the normalized propagation constant δb (52) multiplied by the coefficient $N=(4\Delta R_0/a)^2$ in the weakly guiding limit $\Delta \to 0$.

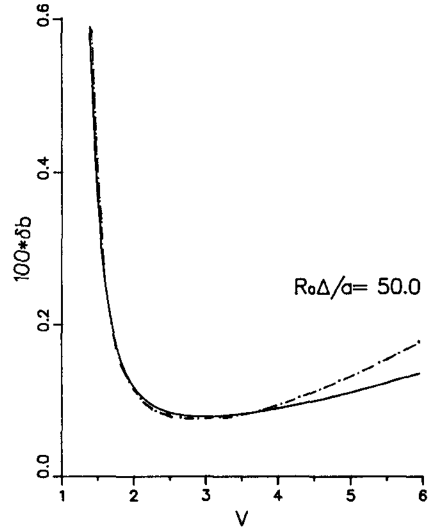

Fig. 3. Change of the normalized propagation constant δb at $R_0\Delta/a=50$ in the weakly guiding limit $\Delta \to 0$. The solid line refers to the asymptotic solution (52) and the dash–dot line to the functional (50).

where $M_1=f^2+g^2$, $M_2=2f'$, $M_3=4f'(t'+t/\rho)\rho$, and $M_4=(t'+t/\rho)^2$. The functions t, f, and g are defined in (47). The prime denotes the first derivative with respect to the argument. The evident advantage of the asymptotic formulation over the iterative method is that (52) does not contain singularities in the integrand, in contrast to the functional (50).

The accuracy of the field expansion (40) depends on the parameter (α/R_0). If this parameter is small enough, indicating a large normalized radius of curvature $R_0\Delta/a$, the first terms in the series are sufficient. On the other hand, with a small radius of curvature more terms are required or, alternatively, the differential equation (39) must be solved directly without any perturbational approach. In that case, the normalized propagation constant is strongly modified by the bend.

Results by using the zero-order and first-order fields (43), (44), (45), and (46) are depicted in Figs. 2 and 3. Fig. 2 shows a general curve for δb calculated from (52). This curve is identical with that given in [45], which has been obtained by applying a transversal field formulation and asymptotic method. In Fig. 3 the asymptotic curve is compared with that from the functional (50). The curves go very close to each other, except at large V value region, where the functional (50) predicts somewhat higher δb values. For a more realistic fiber with a nonzero Δ parameter we have to apply (28) directly. The asymptotic approach in this case is useless because the equation would then be much more complicated than (52). In Table I are summarized δb results at two different Δ values, namely $\Delta=0.010$ and 0.004, and at $R\Delta/a=20$, 50, and 80. As it is seen, δb values are slightly affected by the normalized difference of the dielectric constants.

Some of the values in the table are marked with an asterisk to indicate that here (50) has more than one solution. The origin of these additional solutions is probably related to the strong growth of the δb curves in the small V value region and also to the asymptotic approximation of the field, which for a small radius of curvature should indicate more terms than are used here in the calculations.

TABLE I
VALUES OF CHANGE OF THE NORMALIZED PROPAGATION CONSTANT δb IN THE SINGLE-MODE OPTICAL FIBER AT DIFFERENT VALUES OF THE NORMALIZED RADIUS OF CURVATURE AND AT $\Delta=0.010$, $\Delta=0.040$

	δb							
V	$R_0=\infty$ b_0		$R_0\Delta/a=20$ $\delta b\times10^3$		$R_0\Delta/a=50$ $\delta b\times10^3$		$R_0\Delta/a=80$ $\delta b\times10^4$	
	$\Delta=0.010$	0.004	0.010	0.004	0.010	0.004	0.010	0.004
5.395	.8601	.8593	7.46	7.37	1.48	1.47	5.93	5.86
4.295	.7963	.7961	5.92	5.86	1.04	1.04	4.14	4.10
3.195	.6800	.6806	4.69	4.64	0.78	0.78	3.06	3.02
2.095	.4439	.4448	6.56	7.08	1.02	1.01	3.98	3.94
1.545	.2479	.2465	32.0*	21.7*	3.34	3.10*	12.6	12.5

b_0 values for the straight fiber have been calculated from the functional (33).

V. Application to Anisotropic Fibers

In this section we apply the functional (27) for two different kinds of anisotropic fibers. In the first example we have a fiber with anisotropic core and isotropic cladding, while in the second example both the core and the cladding are anisotropic. One of the fiber anisotropy axes is taken to be in the plane of the bend. The anisotropy is assumed to originate in the mechanical stress, thus producing linear birefringence for which the modes are linearly polarized. The lowest HE_{11} mode is then approximated by (47) with the fields (43), (44) and (45), (46). This approximation can be made provided that the anisotropy is minor, as assumed in the subsequent examples. It is worth noting that this assumption is not inherent in the functional (27), which can treat all kinds of transversely and symmetrically anisotropic waveguides.

A. Circular Step-Index Fiber with Anisotropic Core and Isotropic Cladding

In this waveguide both the core and the cladding are homogeneous but the dielectric function of the former is transversely anisotropic. More exactly, the dielectric dyadic takes on the form

$$\boldsymbol{\epsilon} = \epsilon_2\left[\boldsymbol{E} + \Delta P(\rho)\boldsymbol{\kappa} + (1+\Delta P(\rho))\boldsymbol{u}_s\boldsymbol{u}_s\right] \tag{53}$$

where P is the pulse function (38) and $\boldsymbol{\kappa}$ is the two-dimensional dyadic

$$\boldsymbol{\kappa} = A_x\boldsymbol{u}_x\boldsymbol{u}_x + A_y\boldsymbol{u}_y\boldsymbol{u}_y \tag{54}$$

with constants A_x and A_y. The normalized parameters V and b must be defined as

$$V = k_2 a\sqrt{\Delta} \qquad b = \left(\beta^2 - k_2^2\right)a^2/V^2 \tag{55}$$

because the parameter k_1 is not unique in the present fiber.

To apply the general functional (27) we start from the dyadic $\boldsymbol{M}$, which now can be written as

$$\begin{aligned}\boldsymbol{M} &= \frac{\omega^2\boldsymbol{k}_c^{-2}}{R^2}\\ &= \frac{\omega^2a^2}{R^2V^2}\left\{\boldsymbol{\kappa}P + \left(\frac{1-(R_0/R)^2}{\Delta} - \left((R_0/R)^2b\right)\right)\boldsymbol{E}\right\}^{-1}\\ &= \frac{\omega^2a^2}{V^2}\boldsymbol{D}_1.\end{aligned} \tag{56}$$

The dyadic $\boldsymbol{D}_1$ is the inverse of the two-dimensional dyadic, which by taking the limit $\Delta \to 0$ can be expressed in the following formula:

$$\begin{aligned}\boldsymbol{D}_1 &= \frac{\boldsymbol{u}_x\boldsymbol{u}_x}{R_0^2(PA_x - b_0)F(PA_x, b_0)} + \frac{\boldsymbol{u}_y\boldsymbol{u}_y}{R_0^2(PA_y - b_0)F(PA_y, b_0)}\\ &= \frac{\boldsymbol{D}}{R_0^2}\end{aligned} \tag{57}$$

where the definition (17) for the inverse dyadic has been applied. The function F is defined in (50). Substituting (57), (56), and (53) into (27) gives the functional for the parameter V^2:

$$V^2 = \frac{\int\left(\sqrt{\epsilon_2}\,\nabla e - \sqrt{\mu}\,\boldsymbol{u}_s\times\nabla h\right)\cdot\boldsymbol{D}\cdot\left(\sqrt{\epsilon_2}\,\nabla e - \sqrt{\mu}\,\boldsymbol{u}_s\times\nabla h\right)dS}{\int\left(\epsilon_2e^2 + \mu h^2\right)dS} \tag{58}$$

in the limit $\Delta \to 0$. The integration extends over the entire transverse plane. This equation is closely related to (50) for the isotropic fiber. To see this we take a fiber possessing only one anisotropic parameter, say A_x and $A_y = 1$. Now, for the HE_{11y} mode polarized in the y direction, the vector $(\sqrt{\epsilon}\,\nabla e - \sqrt{\mu}\,\boldsymbol{u}_s\times\nabla h)$ is y directed and when multiplied by the dyadic $\boldsymbol{D}$ gives the functional for the isotropic fiber, (50). Thus, for this mode the fiber behaves as an isotropic one. On the contrary, for the HE_{11x} mode the above vector is directed along the x axis and the functional differs from that of the isotropic fiber by the anisotropic parameter A_x. The functional is now

$$b = A_x f\left(\sqrt{A_x}\,V, P\right) + \delta f\left(\sqrt{A_x}\,V, P\right)/A_x \tag{59}$$

if the isotropic relation is denoted by

$$b = f(V,P) + \delta f(V,P) \tag{60}$$

and the latter terms in (59) and (60) stand for the change of the propagation constant due to the bending. The first terms relate the anisotropic straight fiber to the corresponding isotropic straight fiber. It has been shown that this transformation relation for the straight fibers can be applied for a large variety of inhomogeneous fibers [46].

Equation (59) is valid for all possible parameter values A_x. If the anisotropy is in the y direction, the dispersion relation for the HE_{11y} mode is described by (59) with A_x replaced by A_y. The reason for this symmetry comes from the symmetrical diadic $\boldsymbol{D}$ (57). The effect of the anisotropy parameter A_x is depicted in Fig. 4.

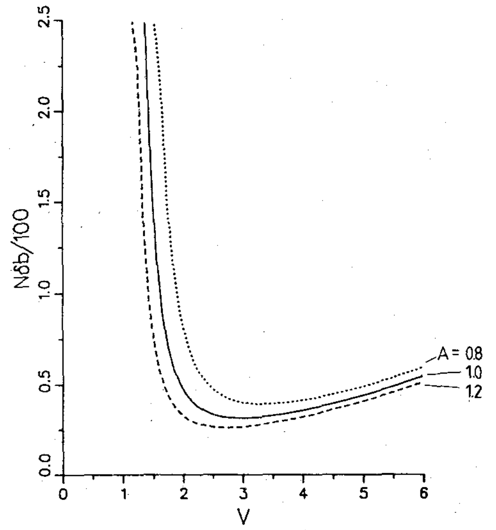

Fig. 4. The effect of the transversal anisotropy, parameter A, on the change of the normalized propagation constant $N\delta b$, where $N = (4\Delta R_0/a)^2$ in the weakly guiding limit $\Delta \to 0$.

The difference between the propagation constants of the orthogonal polarized waves of the same mode is called the birefringence, defined as

$$B = 2\frac{\beta_x a - \beta_y a}{\beta_x a + \beta_y a} = 2\frac{\sqrt{b_x + 1/\Delta} - \sqrt{b_y + 1/\Delta}}{\sqrt{b_x + 1/\Delta} + \sqrt{b_y + 1/\Delta}} \approx \frac{\Delta}{2}\left(b_x - b_y\right). \tag{61}$$

The last expression is valid for $\Delta \to 0$ if b_x and b_y are not very small. If the fiber has only a minor anisotropy, the propagation constants of the two polarizations do not

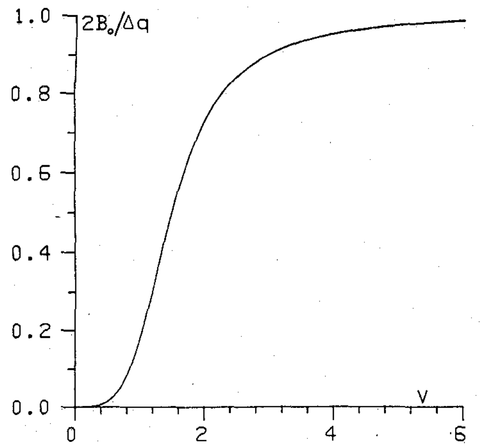

Fig. 5. Normalized birefringence $2B_0/\Delta q$ for the step-index fiber with anisotropic core and isotropic cladding in the limit of weak guidance and perturbational anisotropy [36].

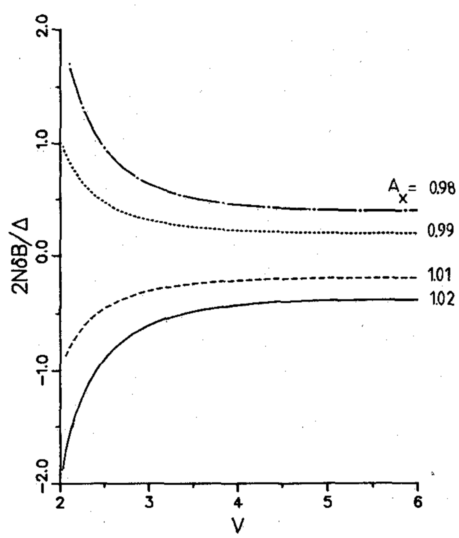

Fig. 6. Effect of the bending on the normalized birefringence $2N\delta B/\Delta$ of the step-index optical fiber with anisotropic core and isotropic cladding in the limit of weak guidance and perturbational anisotropy. The coefficient $N = (4\Delta R_0/a)^2$.

differ by much. In that case the birefringence can be calculated by using the fields derived for the isotropic fiber. If the anisotropy is perturbational in one direction, say x, the birefringence can be expressed in the following form for $A_x = 1 + q$:

$$B \approx \frac{\Delta q}{2}\left(b_0(V) + \frac{1}{2}Vb_0'(V)\right) + \frac{\Delta}{2}(\delta b_x - \delta b) = B_0 + \delta B \tag{62}$$

where the asymptotic formula of the propagation constant (49) has been adopted. The prime denotes the derivative of b_0 with respect to V. The first part of the equation refers to the straight fiber, whereas the latter part is an additional term due to the bend. This can be calculated asymptotically from (52) according to the transformation (59). If a more exact analysis is required, the dispersion curve must be evaluated using the functional (50) in (59) and the birefringence from (61). In Fig. 5 the birefringence of the

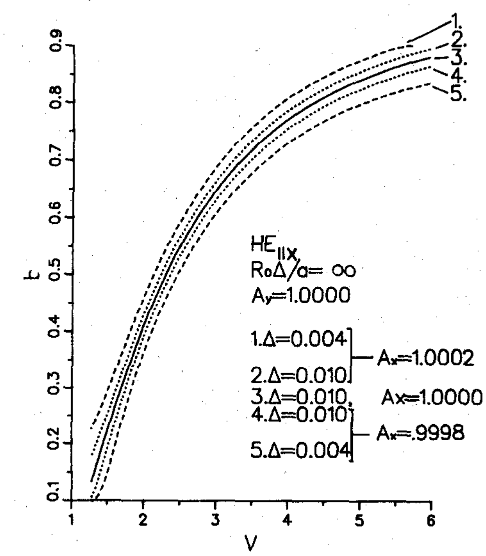

Fig. 7. Dispersion relations of a straight step-index fiber with anisotropic core and cladding for various Δ and anisotropy parameter values.

straight fiber has been plotted by using the analytic dispersion equation [47]

$$b_0(V) = 1 - \left\{(1+\sqrt{2})/\left(1+(4+V^4)^{1/4}\right)\right\}^2. \tag{63}$$

The influence of the bending on the birefringence is depicted in Fig. 6 for various anisotropy parameter values.

B. Circular Step-Index Fiber with Anisotropic Core and Cladding

As a second example we consider a bent step-index anisotropic fiber with the dyadic function

$$\boldsymbol{\epsilon} = \epsilon_2(1+\Delta P(\rho))(\boldsymbol{\kappa} + \boldsymbol{u}_s\boldsymbol{u}_s). \tag{64}$$

Inserting $\boldsymbol{\kappa}$ from (54) into (64) and separating orthogonal dielectric constants in the transverse plane give

$$\epsilon_x = \epsilon_2(1+\Delta P(\rho))A_x \qquad \epsilon_y = \epsilon_2(1+\Delta P(\rho))A_y \tag{65}$$

where $P(\rho)$ equals the pulse function.

The analysis of this fiber cannot be transformed from that of the isotropic one; hence we must work with the general functional (27). Here we again assume the anisotropy to be small enough that the fields of the HE_{11} mode can be approximated by (47) with (43) and (45). This restriction is not related to the functional (27), which was derived for arbitrary anisotropic relations.

To start with this example we consider first a straight anisotropic fiber, applying the functional (32) with the fields (43) and (44). The dispersion curves at various Δ and anisotropy parameter values are depicted in Fig. 7. The curves, which have been calculated for the HE_{11x} mode with the x-directed anisotropy, are identical for the HE_{11y} mode and the y-directed anisotropy. The results are within the reading accuracy as are those given in [36]. This confirms our previous calculations based on application of elementary trial functions.

To proceed, we bend the fiber by the normalized radius $R_0\Delta/a = 50$ and apply the functional (27), yielding Figs. 8

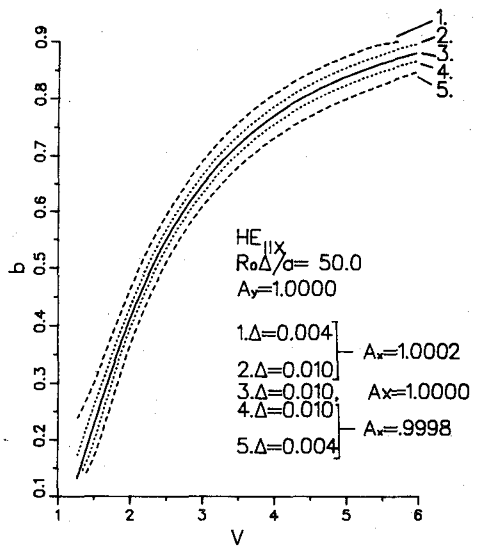

Fig. 8. Dispersion relation of HE_{11x} mode in the curved fiber with anisotropic core and cladding for various Δ and x-directed (in the plane of the bend) anisotropy parameter values.

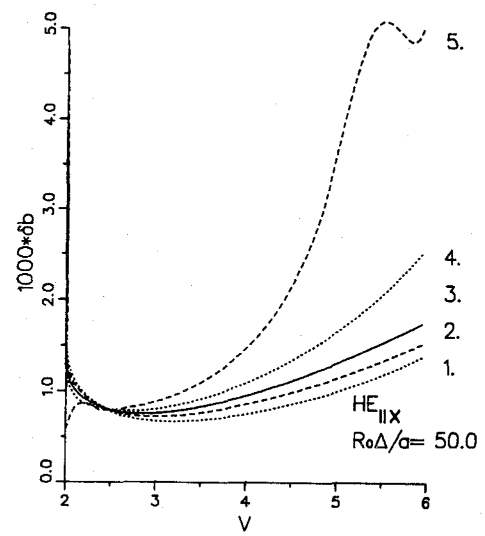

Fig. 10. Change of the normalized propagation constant due to the bend in the optical fiber with x-directed anisotropy in the core and in the cladding. The numbers refer to the corresponding curves in Fig. 8.

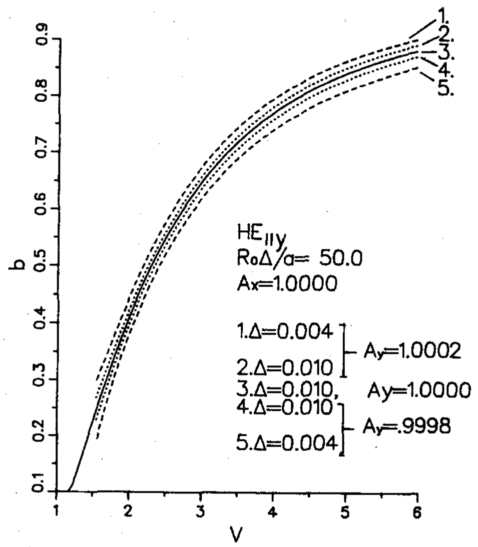

Fig. 9. Same as Fig. 8 for HE_{11y} mode and y-directed anisotropy.

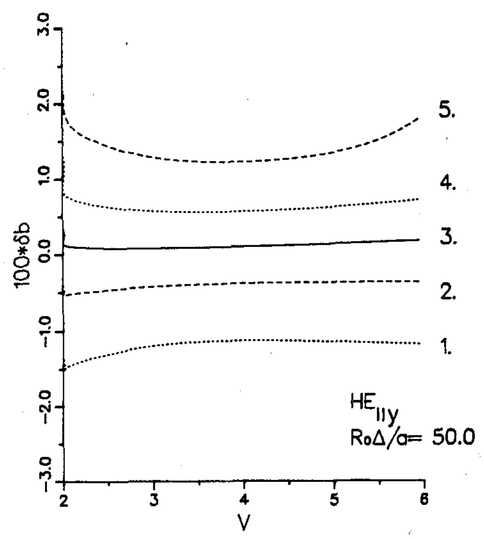

Fig. 11. Same as Fig. 10 for y-directed anisotropy. The numbers refer to the curves given in Fig. 9.

and 9 for HE_{11x} and HE_{11y} modes, with the direction of anisotropy along the x and the y axis, respectively. The symmetry existing in the straight fiber now disappears. This is clearly observed in Figs. 10 and 11 where the change of the dispersion curve due to the bending is shown at both polarizations. The numbers refer to the corresponding curves in Figs. 8 and 9. The solid lines denoting the isotropic curves are the same for both polarizations, whereas the effect of the bend is more pronounced in the HE_{11y} mode.

The effect of the anisotropy, and thus birefringence, is studied in Figs. 12, 13, and 14. The birefringence of the straight fiber (Fig. 12) is almost the same, excluding curve no. 5, as that in the curved fiber at the HE_{11x} mode (Fig. 13). This indicates that here the x-directed anisotropy is strong enough to prevent additional changes due to the bend. The curves of the HE_{11y} mode differ from those in Figs. 12 and 13 and thus are influenced by the bend.

VI. Conclusions

A stationary functional for anisotropic curved waveguides has been derived. The functional is general in that it includes longitudinal and/or transversal anisotropy and homogeneous or nonhomogeneous media. It can be applied to closed waveguides with an arbitrary radius of curvature and to open waveguides with a slight radius of curvature. As an example of the method a curved step-index round optical fiber has been studied. First, the differential equation for the longitudinal electric field has been solved, giving field equations for the straight fiber and the lowest order corrections to the real part of the propagation constant due to the bending. These solutions are then taken for the trial fields for isotropic and anisotropic fibers. The

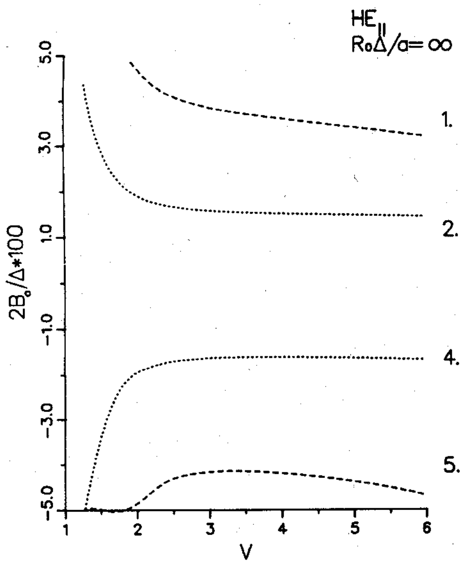

Fig. 12. Normalized birefringence of a straight step-index optical fiber with anisotropic core and cladding. The numbers refer to the curves in Fig. 7.

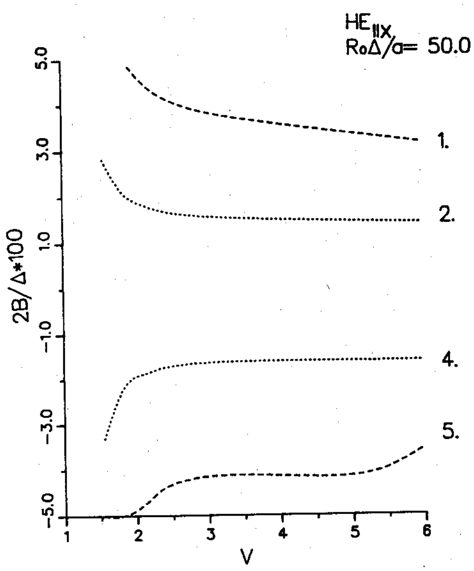

Fig. 13. Same as Fig. 12 for curved anisotropic fiber and for the HE_{11x} mode. The numbers refer to the curves in Fig. 8.

functional for the isotropic step-index fiber has been derived under the weakly guiding assumption. It has been calculated asymptotically and iteratively, where in the former singularities in the integrand can be avoided. Results from two different methods are seen to be very close to each other. It is also noted that the change in the dispersion curve due to the bend is the same for orthogonal polarized HE_{11} modes. Application of the theory to a fiber possessing perturbational anisotropy in the core related to the isotropic cladding was seen to lead to transformation equations for the dispersion characteristics and analytical expressions for the birefringence. In the fiber with both the core and the cladding anisotropic, dispersion curves and birefringence were seen to be dependent on the direction of anisotropy, whereas in the former anisotropic fiber,

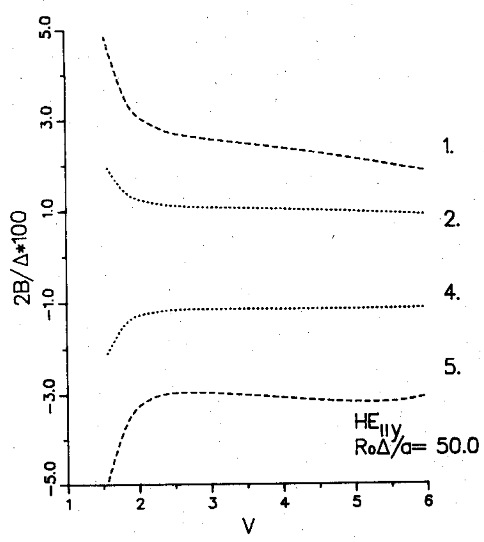

Fig. 14. Same as Fig. 13 for HE_{11y} mode. The numbers refer to the curves in Fig. 9.

these characteristics were invariant with respect to the direction of anisotropy in the weakly guiding limit.

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